Sec. 6.5 Combining Transformations

Ordering Horizontal and Vertical Transformations

For nonzero constants A, B, h and k, the graph of the function

$$y = A\,f(B\,(x-h)) + k$$

is obtained by applying the transformations to the graph of $f(x)$ in the following order:

- Horizontal stretch/compression by a factor of $1/|B|$
- Horizontal shift by h units
- Vertical stretch/compression by a factor of $|A|$
- Vertical shift by k units

If $A < 0$, follow the vertical stretch/compression by a reflection about the x-axis.
If $B < 0$, follow the horizontal stretch/compression by a reflection about the y-axis.

Ex. Graph the following functions by hand, verifying each step.

a. $f(x) = \frac{1}{3}(x+3)^2$

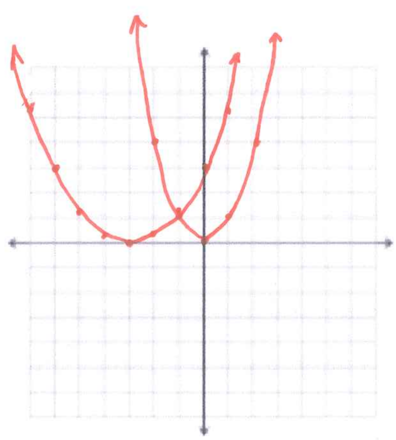
- HORIZONTAL TRANSLATION LEFT 3
- VERTICAL COMPRESSION SF $\frac{1}{3}$

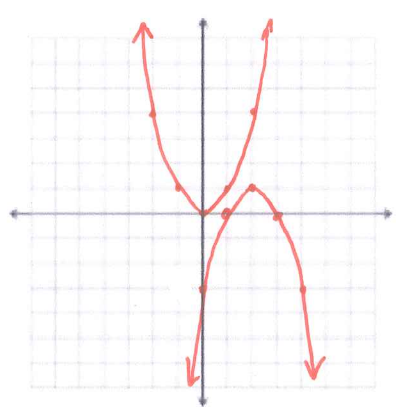
b. $f(x) = -(x-2)^2 + 1$

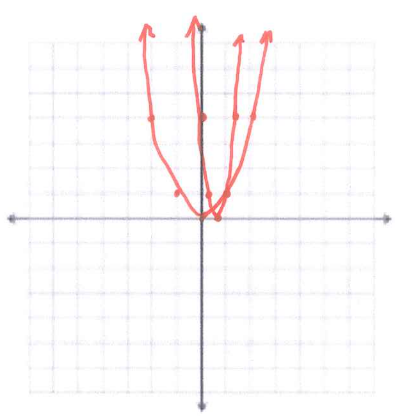
HORIZONTAL TRANSLATION RIGHT 2
REFLECTION VERTICALLY ACROSS X-AXIS
VERTICAL TRANSLATION UP 1

c. $f(x) = (3x-2)^2$ $f(x) = (3(x-\frac{2}{3}))^2$

HORIZONTAL COMPRESSION SF $\frac{1}{3}$
HORIZONTAL TRANSLATION RIGHT $2/3$







x	y
0	4
$\frac{1}{3}$	1
$\frac{2}{3}$	0
1	1
$1\frac{1}{3}$	4

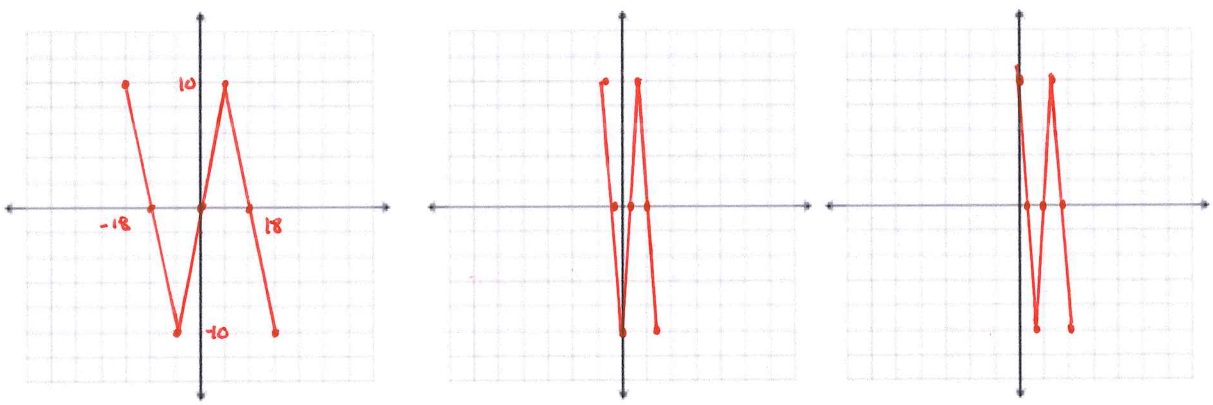
Ex. Graph the following table.

X	(a)		(b)		F(x)
-18	-12	-4	-6	0	10
-12	-6	-2	-4	2	0
-6	0	0	-2	4	-10
0	6	2	0	6	0
6	12	4	2	8	10
12	18	6	4	10	0
18	24	8	6	12	-10

a. Graph the function that is obtained by first shifting the graph horizontally right by 6 units and then compressing horizontally by a factor of 1/3. Give a formula for this function. $f(x-6) \Rightarrow f(3x-6)$ $\{$Same as $f(3(x-2))\}$

b. Graph the function that is obtained by first compressing horizontally the graph by a factor of 1/3 and then shifting it horizontally to the right 6 units. Give a formula for this function. $f(3x) \Rightarrow f(3(x-6))$ $\{$Same as $f(3x-18)\}$

c. Compare the graphs. How are they related?



Ex. (a) Rewrite the function $y = f(2x-6)$ in the form $y = f(B(x-h))$.

$y = f(2(x-3))$

(b) Use the result to describe the graph of $y = f(2x-6)$ as the result of first applying a horizontal stretch or compression to the graph of f and then applying a horizontal shift. What is the stretch/compression factor? What is the shift?

Horizontal compression sf $\frac{1}{2}$
Horizontal shift right 3

Would be the same as a horizontal shift right 6 and then a horizontal compression sf $\frac{1}{2}$.

Ex. The graph of g is found by shifting the graph of f right four units, then stretching horizontally by a scale factor of five, then reflecting it vertically across the x-axis, then shifting it up three units, then reflecting it horizontally across the y-axis, and finally stretching it vertically by a scale factor of 4. Write an equation for g in terms of f.

$g(x) = f(x-4)$
$= f(\frac{1}{5}x-4)$
$= -f(\frac{1}{5}x-4)$
$= -f(\frac{1}{5}x-4)+3$
$= -f(-\frac{1}{5}x-4)+3$
$= 4(-f(-\frac{1}{5}x-4)+3)$
$g(x) = -4f(-\frac{1}{5}x-4)+12$

Ex. If (16, -12) is a point on the graph of f, find the new point on the graph of $g(x) = -2f(-4x-12)+3$.

$-2f(-4(x+3))+3$

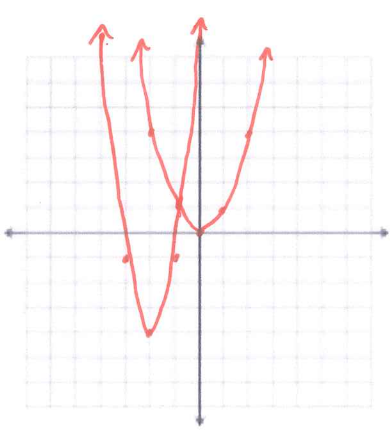
Horizontal compression sf $\frac{1}{4}$	(4, -12)
Horizontal reflection across y-axis	(-4, -12)
Horizontal Translation left 3	(-7, -12)
Vertical Stretch SF 2	(-7, -24)
Vertical Reflection across x-axis	(-7, 24)
Vertical Translation up 3	(-7, 27)

Ex. (a) Let $y = 3(x+2)^2 - 4$. Determine the values of A, B, h, and k when y is put in the form $y = Af(B(x-h)) + k$ with $f(x) = x^2$. List the transformations applied to $f(x) = x^2$ to give $y = 3(x+2)^2 - 4$.

$A = 3$ $B = 1$ $h = -2$ $K = -4$

Horizontal translation left 2
Vertical stretch sf 3
Vertical translation down 4

(a) Sketch a graph of $y = 3(x+2)^2 - 4$, labeling the vertex.



Horizontal translation left 2
Vertical stretch sf 3
Vertical translation down 4

HW: pg 261-264 # 3-45 (m/3)